Numeric Response

Questions Definite Integration

Q.1 If $f(x)=\frac{e^x}{1+e^x}, I_1=\int_{f(-a)}^{f(a)} xg(x(1-x))dx$ and $I_2=\int_{r(-a)}^{r(a)} g(x(1-x))dx$, then find the value of $\frac{I_2}{I_1}$.

Q.2 Find the value of $\int_{-2}^{2}|[x]|$dx (where [,]$denotesgreatestintegerfunction$)

Q.3 Evaluate: $\int_{-1}^{1}\log\left(x+\sqrt{x^2+1}\right)dx$

Q.4 Evaluate: $\int_{\pi}^{10\pi}|\sin\ x|dx$

Q.5 If $\int_{0}^{\pi/5}\cos^3\ 4\theta d\theta$ is equal to $\frac{2}{\lambda}$ then find λ.

Q.6 Evaluate: $\int_{-\pi/2}^{\pi/2}\frac{\cos\ x}{1+e^x}dx$

Q.7 If $I_n=\int_{0}^{\pi/4}\tan^n\ xdx$ and $\mathrm{Is}_s+\mathrm{I}_6=\frac{1}{\lambda}$ then find λ.

Q.8 If $\int_{0}^{1}\frac{\tan^{-1}\ x}{1+x^2}dx=\frac{\pi^2}{k}$ then find value of k.

Q.9 Find the value of integral $\int_{-1}^{1}|1-x|dx$.

Q.10 If $\int_{0}^{a}\frac{x^7dx}{\sqrt{a^2-x^2}}=\frac{k}{\lambda}k^i$ then find value of $\lambda-k$.

Q.11 If $\int_{0}^{\pi/2}\sin^4\ x\cos^3\ xdx=\frac{7\pi}{k}$ then find value of k.

Q.12 Evaluate: $\int_{0}^{200x}\sqrt{\frac{1-\cos\ 2x}{2}}dx$

Q.13 If $\int_{0}^{\pi/2}\frac{\sqrt{\sin\ x}}{\sqrt{\sin\ x}+\sqrt{\cos\ x}}dx=\frac{\pi}{\lambda}$ then find the value of λ.

Q.14 Find the value of $\int_{0}^{2014}\frac{2^x}{2^x+2^{2014-x}}dx$.

Q.15 If $\int_{0}^{1}x^6\sqrt{1-x^2}dx=\frac{\lambda\pi}{k}$ then find $\lambda+k$.

ANSWER KEY

1. 2.00	**2.** 4.00	**3.** 0.00	**4.** 18.00	**5.** 12.00	**6.** 1.00	**7.** 7.00
8. 32.00	**9.** 2.00	**10.** 19.00	**11.** 2048.00	**12.** 400.00	**13.** 4.00	**14.** 1007.00
15. 261.00						

Hints & Solutions

1. Note : f(a) + f(–a) = 1

$$I_1 = \int_{f(-a)}^{f(a)} xg(x(1-x))dx$$

$$I_1 = \int_{f(-a)}^{f(a)} (1-x)g((1-x)x)dx$$

$$\Rightarrow I_1 + I_1 = \int_{f(-a)}^{f(a)} g(x(1-x))dx$$

$$\Rightarrow 2I_1 = I_2$$

$$\Rightarrow \frac{I_2}{I_1} = 2$$

2. $$I = \int_{-2}^{-1} |-2|\,dx + \int_{-1}^{0} |-1|\,dx + \int_{0}^{1} |0|\,dx + \int_{1}^{2} |1|\,dx$$

$$\Rightarrow 2(x)_{-2}^{-1} + 1.(x)_{-1}^{0} + 0 + (x)_1^2$$

= 4

3. as $\log(x + \sqrt{1+x^2})$ is an odd function. So

$$\int_{-1}^{1} \log(x + 1\sqrt{1+x^2})\,dx = 0$$

4. $$\int_{\pi}^{10\pi} |\sin x|\,dx = 9\int_0^{\pi} |\sin x|\,dx$$

$$= 9\int_0^{\pi} \sin x\,dx = 18$$

5. $4\theta = t$

$$\therefore I = \frac{1}{4}\int_0^{\pi/2} \cos^3 t\,dt$$

$$= \frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$$

6. Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

We get $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^{-x}} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^{-x}}dx$

$$\therefore 2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\ dx = 2$$

$$\Rightarrow I = 1$$

7. $$I_n = \int_0^{\pi/4} \tan^n x\,dx$$

$$= \int_0^{\pi/4} \tan^{n-2} x(\sec^2 x - 1)dx$$

$$= \frac{1}{n-1} - I_{n-2}$$

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1}$$

$$\Rightarrow I_8 + I_6 = \frac{1}{8-1} = \frac{1}{7}$$

8. $$I = \int_0^1 \frac{\tan^{-1} x}{1+x^2}dx$$

Put $\tan^{-1}x = t$ also, when x = 0, t = 0

$\frac{1}{1+x^2}dx = dt$ when x = 1, t = π/4

$$I = \int_0^{\pi/4} t\,dt$$

$$I = \left[\frac{t^2}{2}\right]_0^{\pi/4}$$

$$I = \frac{(\pi/4)^2}{2} - 0$$

$$I = \frac{\pi^2}{32}$$

9. $I = \int_{-1}^{1} |1-x|\,dx$

$I = \int_{-1}^{1} (1-x)\,dx$

$I = \left(x - \frac{x^2}{2}\right)_{-1}^{1}$

$I = \left(1-\frac{1}{2}\right) - \left(-1-\frac{1}{2}\right)$

$I = 2$

10. $I = \int_{0}^{\pi/2} \frac{a^7 \sin^7 \theta . a \cos\theta\, d\theta}{a\cos\theta}$

put x/a = sin θ

dx = a cos θ dθ

$I = a^7 . \frac{6.4.2}{7.5.3.1} = \frac{16}{35} a^7$

11. $I = \int_{0}^{\pi/2} \sin^4 x \cos^8 x\, dx$

$= \frac{(4-1)(4-3)\times(8-1)(8-3)(8-5)(8-7)}{12.10.8.6.4.2.1} \frac{\pi}{2}$

$I = \frac{7\pi}{2048}$

12. $I = \int_{0}^{200\pi} \sqrt{\frac{1-\cos 2x}{2}}\,dx = \int_{0}^{200\pi} |\sin x|\,dx$

$= 200\int_{0}^{\pi} |\sin x|\,dx = 200\int_{0}^{\pi} \sin x\, dx$

$= 200 \times 2 = 400$

13. Let $I = \int_{0}^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$

Here the lower limit is zero hence, we can replace x by (a – x) i.e. by π/2 – x

$\therefore I = \int_{0}^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)} + \sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$

$= \int_{0}^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx$

Adding $2I = \int_{0}^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

14. $\int_{0}^{2014} \frac{2^x}{2^x + 2^{2014-x}}\,dx = \frac{2014}{2} = 1007$

15. Let x = sin θ ⇒ dx = cos θ dθ

$\int_0^{\pi/2} \sin^6 \theta . \cos^2 \theta\, d\theta$

$= \frac{5\cdot3\cdot1\cdot1}{8\cdot6\cdot4\cdot2} \times \frac{\pi}{2} = \frac{5\pi}{256}$